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THE VACUUM CATASTROPHE REVISITED: WHEN THE QUANTUM VACUUM DOES NOT CONTRIBUTE TO COSMOLOGICAL EQUILIBRIUM

Application to an Expanding Hypersphere

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Claudio Marchesan

Chemical Engineer (Retired)

e-mail: clmarchesan@gmail.com

ABSTRACT

In this short note, we show that the issue of dynamical vacuum energy can be consistently addressed within a framework based on Dynamics on an Expanding Hypersphere, as an alternative geometrical description to the standard FLRW approach. In a finite S^3 geometry, the renormalized electromagnetic vacuum energy depends only on geometric variations and results in a subdominant correction to the cosmological dynamics.

DISCUSSION

The resolution of the vacuum catastrophe emerges from two structural features of the model: its formulation within a finite, curvature-defined spatial geometry, which renders the vacuum spectrum intrinsically discrete and removes the need for any ultraviolet cutoff, and the absence of an effective cosmological constant, since the renormalized vacuum contribution—already negligible in magnitude—cancels exactly under hyperspherical equilibrium.

In this work we examine the structure of the quantum vacuum in a finite S^3 cosmological framework, adopting an interpretation in which only geometry-dependent vacuum variations are physically meaningful [1]. In the model described in [2], we distinguish between an intergalactic sector, in which the only dynamical field is the Cosmic Microwave Background (CMB), and localized regions containing matter and gravitational inhomogeneities, treated as perturbative deviations from the homogeneous hyperspherical background. In a closed S^3 geometry, the field modes are discrete and determined by the global curvature scale rather than by an infinite-volume continuum.

As a consequence, the renormalized [3] vacuum energy depends solely on geometric variations of the hypersphere and yields a subdominant contribution compared to the CMB radiation pressure. In matter-dominated regions, the vacuum contribution remains subleading, consistent with Einstein's original formulation of the field equations without a cosmological constant [4].

Within this framework, the equilibrium condition of the S^3 hyperbubble is governed entirely by the CMB field, while the quantum vacuum plays no dynamical role. This perspective offers a natural resolution to the vacuum energy problem and provides a coherent physical interpretation of intergalactic space as a CMB-dominated vacuum on a finite S^3 geometry.

[1] – The Casimir Effect and the Quantum Vacuum

[2] – This model is fundamentally simple, rooted in the geometry of the 3-sphere. It belongs to the field of alternative cosmology, though the core concept of the hypersphere has an illustrious origin, being attributed to Einstein himself.

From a macroscopic perspective, the model relies on the physical laws and principles that we apply to the real world; from a microscopic viewpoint, it considers only the particles currently known to the Standard Model. While not exhaustive, I have endeavored to describe it accurately through the following series of publications:

- Dynamics on an Expanding Hypersphere: Reassessing the Cosmological Principle in light of the CMB
- The Vacuum Catastrophe revisited: when the quantum vacuum does not contribute to cosmological equilibrium
- The Aporia of Reversibility in Cosmological Expansion
- Reevaluating the Necessity of Dark Matter and Dark Energy within Cosmological Models
- A new perspective on Hubble's law through a four-dimensional spatial model

I have termed it the '4-Sphere' model. This choice was made partly to distinguish it from other mathematical treatments, but also because, while setting up a stellar map for the model, I encountered 2D, 3D, and 4D geometric figures intricately intertwined. In that moment, I realized that the mathematical term '3-sphere'—though technically correct—did not fully capture the profound 4-dimensional spatial reality of the object I was observing.

[3] – *Renormalization* is a technical procedure used in quantum field theory to make sense of quantities that would otherwise appear infinite.

The key idea is that the *absolute* value of the quantum vacuum energy is not physically measurable. What experiments can access are only differences, or changes, in that energy — for example, between two configurations of space, or between two nearby points, or before and after an interaction.

Because only these differences matter, the formalism allows us to subtract the unphysical part of the vacuum energy — the part that would diverge — and retain only the finite, physically meaningful contribution. This does not mean "throwing away infinities by hand", nor introducing an arbitrary cutoff. Instead, it reflects the empirical fact that any measurement apparatus interacts only with *variations* of the quantum fields, never with their absolute baseline.

A classic example is the Casimir effect: two conducting plates modify the vacuum fluctuations between them, and the observable force comes entirely from the difference between two vacuum states. Each individual vacuum energy would diverge if taken alone, but the *difference* is finite and measurable.

In short:

- The vacuum energy in QFT contains terms that grow without bound.
- These terms do not correspond to observable physics.
- Renormalization systematically isolates the finite changes that do correspond to observables.
- No arbitrary cutoff is required as long as one works with differences of vacuum energies, which are naturally finite.

This is why renormalization is compatible with our approach: by focusing on local differences of the vacuum energy density in a slowly varying background (such as an S^3 embedded in the 4-sphere), you automatically select the meaningful, finite contributions, just like in the Casimir mechanism.

[4] – The position adopted in this model regarding the absence of a cosmological constant Λ is discussed in detail in: Reevaluating the Necessity of Dark Matter and Dark Energy within Cosmological Models

Energy-balance equation with quantum vacuum contribution

In this model, the only energetic component present in the intergalactic vacuum is the cosmic microwave background (CMB). The aim of this note is to verify that the quantum contribution associated with the electromagnetic zero-point energy does not introduce divergences that would be incompatible with the energy-balance equation governing the expanding S^3 surface, and that its effective pressure remains negligible.

The considerations developed so far allow us to reconsider the balance equation, described in [5], that governs the dynamics of the universe:

$$\frac{\partial(c^2\rho\,\delta V)}{\partial t}\,dt = \delta w - p\,\frac{\partial(\delta V)}{\partial t}\,dt,$$

where δw still represents the work done to keep the universe in its shape. Here, the density ρ and pressure p terms do not correspond solely to the CMB; they must also incorporate the intrinsic contribution of the electromagnetic vacuum.

In equilibrium, the total right–hand side must tend to zero. The vacuum terms are therefore introduced only to verify that it does not destabilize this balance.

[5] – Dynamics on an Expanding Hypersphere: Reassessing the Cosmological Principle in light of the CMB

Negligibility of Quantum Vacuum density and pressure on the S^3 hypersphere

In the present cosmological framework, where the universe is modeled as a 3-sphere (S^3) with a radius $R = ct \approx$ 13.8 billion light-year (Gyr), the contribution of quantum vacuum fluctuations to the overall pressure balance is demonstrably negligible.

By adopting the standard heat-kernel renormalization via the zeta-function prescription—as established by Dowker and Critchley (1976) in their seminal work *"Scalar effective Lagrangian in de Sitter space"* (Phys. Rev. D)—the renormalized electromagnetic vacuum energy density [6] scales as

$$\rho_{\text{vac}}^{(S^3)} \propto \frac{\hbar c}{R^4}$$

In contrast, our model assumes the conservation of the Cosmic Microwave Background (CMB) energy, leading to a radiation density that scales as

$$\rho_{\mathrm{CMB}}^{(S^3)} \propto \frac{\hbar c}{R^3}$$

Even the pressure scales this way, it is obtained from $V = 2\pi^2 R^3$

$$E_{\mathrm{vac}}^{\mathrm{ren}}(R) \propto \frac{\hbar c}{R} \qquad p_{\mathrm{vac}} = -\frac{dE_{\mathrm{vac}}^{\mathrm{ren}}}{dV} \qquad [7]$$

this yields the scaling

$$p_{\mathrm{vac}} \propto -\frac{\hbar c}{R^4}.$$

Given the current cosmological radius, the ratios $\rho_{\mathrm{vac}}^{(S^3)}/p_{\mathrm{CMB}}^{(S^3)}$ and $p_{\mathrm{vac}}^{(S^3)}/p_{\mathrm{CMB}}^{(S^3)}$ are approximately 10^{-107}, ensuring that the vacuum pressure remains many orders of magnitude below the CMB threshold

$$p_{\mathrm{CMB}}^{(S^3)} \approx 4.2 \cdot 10^{-14}\mathrm{Pa}$$

Consequently, the quantum vacuum exerts no significant influence on the linear expansion dynamics, allowing the $R = ct$ trajectory.

[6] – While the cited work focuses on de Sitter spacetime, the mathematical apparatus for the heat-kernel regularization and the subsequent zeta-function renormalization is directly applicable to the S^3 spatial manifold. In this context, we utilize their derivation of the finite, renormalized vacuum energy residues specific to the hyperspherical geometry, which allows for a consistent treatment of the EM vacuum without introducing a cosmological constant.

[7] – For any isotropic state, including the quantum vacuum, the pressure is defined as the mechanical response of the energy to changes in the volume. Therefore,

$$p_{\mathrm{vac}} = -\frac{dE_{\mathrm{vac}}^{\mathrm{ren}}}{dV},$$

in direct analogy with the thermodynamic identity $dE = -p\, dV$, which remains valid for the renormalized vacuum energy since it depends only on variations with respect to the volume, not on its absolute value.

Off-shell modes and vacuum energy interpretation

In our model, the expansion step dR within the S^3 geometry acts as a geometric source J which is capable of creating or destroying particles [8]; in its absence, the quantum state in the interval $[R, R + dR]$ and over the time range $-\infty < \mathrm{t} < +\infty$ contains only off-shell particles:

If $|0_-\rangle$ denotes the no-particle state for $t = -\infty$ and $|1_{\mathrm{p}}\rangle$ denotes the one particle on-shell state with momentum p at any time t, then:

$$\langle 1_{\mathrm{p}} | 0_- \rangle^{j=0} = 0$$

is the amplitude for creating the particle described above.

However, as demonstrated by our calculations using the scalar effective Lagrangian in de Sitter space, this source generates only a negligible density of on-shell particles. This implies that the vacuum energy remains almost entirely dominated by off-shell modes, which do not contribute to the asymptotic curvature. This result provides a solid QFT foundation for treating the intergalactic vacuum as gravitationally inert, consistent with the kinematic approximation of Special Relativity.

These off-shell field modes correspond to internal configurations in the quantum propagator rather than to asymptotic physical particles, and therefore do not admit a particle interpretation. Their contribution is encoded in the short-distance structure of the propagator and is responsible for the divergent, geometry-independent terms removed by renormalization.

For physical on-shell excitations, energy, momentum, and mass are related by the relativistic dispersion relation

$$E^2 = (pc)^2 + (mc^2)^2,$$

which expresses the mass-shell constraint characterizing observable particles.

Off-shell modes, by contrast, are not subject to this constraint. They do not admit a particle interpretation with well-defined relativistic energy-momentum relations and are not directly observable as free excitations. For this reason, the identity $E = mc^2$ is not applicable to the formal zero-point energy contributions associated with these modes.

Within quantum field theory, only differences in vacuum energy between distinct geometric configurations are physically meaningful. This is explicitly illustrated by Casimir-type effects, where measurable forces arise from changes in boundary conditions rather than from the absolute value of the vacuum energy.

In semiclassical gravity, a homogeneous vacuum contribution would formally enter Einstein's equations through the vacuum expectation value of the stress-energy tensor,

$$\langle \mathrm{T}_{\mu\nu} \rangle = -\mathrm{E}g_{\mu\nu},$$

thereby behaving as an effective cosmological constant term.

[8] – In his book *"Quantum Field Theory: The Why, What and How"*, Padmanabhan formalizes the 'disturbance' of the vacuum through the response of the field to an external source J.

Consistency conditions in this hyperspherical cosmology

In the S^3 framework considered in this work, the renormalized, geometry-dependent variations of the electromagnetic vacuum are in principle included in the dynamical balance of the expanding bubble. However, under hyperspherical equilibrium these contributions cancel out, leaving no residual uniform term proportional to $g_{\mu\nu}$.

As a consequence, no effective cosmological constant arises from the electromagnetic sector within this restricted model, since any residual vacuum contribution would in any case be negligibly small. This outcome is fully consistent with Einstein's original formulation of the field equations without a cosmological term Λ, in which spacetime curvature is sourced solely by dynamical energy–momentum distributions.

Since CMB gravity has been negligible for the last ∼10 Gyr, Special Relativity (SR) provides an excellent approximation for intergalactic dynamics. The cancellation of renormalized vacuum contributions ensures that the quantum vacuum does not introduce additional curvature, justifying a flat kinematic framework. Consequently, the absence of a cosmological term Λ allows for a formal decoupling between the background metric and galactic recession, as sought by our model [9].

We have not examined the intra-galactic region. It is a significant empirical observation that neither the global expansion of the universe nor the effects of the cosmological constant Λ have ever been detected within gravitationally bound systems. Within these domains, the gravitational coherence of the local mass distribution effectively decouples [10] the system from the global S^3 expansion, supporting the hypothesis that matter introduces localized curvature variations on the manifold's surface. These perturbations remain confined and non-destructive, preserving the structural integrity of bound systems against the background cosmological tension [5].

This observational gap supports our decision to exclude Λ from the field equations, as its inclusion would introduce a universal term for a phenomenon that remains undetected in the most precisely measured gravitational environments available to us.

[9] – In this regard see: A new perspective on Hubble's law through a four-dimensional spatial model

[10] – This decoupling remains evident even at the scale of the Local Group, where gravitational interaction between the Milky Way and M31 overrides the global expansion, as demonstrated by the observed blueshift of Andromeda. The lack of any detectable Λ-induced acceleration within these virialized structures suggests that such a term is redundant for describing the physical reality of bound matter.